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Exact solutions for Wick-type stochastic coupled Kadomtsev–Petviashvili equations

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Abstract

The coupled Kadomtsev–Petviashvili (KP) equations with variable coefficients and Wick-type stochastic coupled KP equations are investigated. Exact solutions are shown using the Hermite transform, the homogeneous balance principle and the F-expansion method.

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1. Introduction

In this paper, we shall investigate the periodic wave solutions of the coupled Kadomtsev–Petviashvili (KP) equations with variable coefficients

$$\begin{cases} [u_t + a(t)uu_x + b(t)vv_x - h(t)u_{x^3}]_x + g(t)u_{y^2} = 0, \\ [v_t + h(t)uv_x - h(t)v_{x^3}]_x + g(t)v_{y^2} = 0, \end{cases} \tag{1.1}$$

where $a(t)$, $b(t)$, $g(t)$ and $h(t)$ are bounded or integrable functions on $\mathbb{R}_+$ and satisfy

$$b(t) \neq 0, \qquad h(t) \neq 0 \quad \text{and} \quad a(t) - h(t) = -\alpha b(t), \qquad g(t) = \beta h(t), \tag{1.2}$$

where $\alpha > 0$ and β are constants.

If the problem is considered in a random environment, we can get random coupled KP equations. In order to give the exact solutions of random coupled KP equations, we only consider this problem in a white noise environment, that is, we consider the problem in the white noise space and study Wick-type stochastic KP equations. We shall investigate the following Wick-type stochastic coupled KP equations

$$\begin{cases} [U_t + A(t) \diamond U \diamond U_x + B(t) \diamond V \diamond V_x - H(t) \diamond U_{x^3}]_x + G(t) \diamond U_{y^2} = 0, \\ [V_t + H(t) \diamond U \diamond V_x - H(t) \diamond V_{x^3}]_x + G(t) \diamond V_{y^2} = 0 \end{cases} \tag{1.3}$$

and give their exact solutions, where '$\diamond$' is the Wick product on the Kondratiev distribution space $(\mathcal{S})_{-1}$, namely, the linear space $(\mathcal{S})_{-1}$ is defined in the second section; $A(t)$, $B(t)$, $G(t)$ and $H(t)$ are $(\mathcal{S})_{-1}$-valued variables and satisfy the following conditions:

$$B(t) \neq 0, \qquad H(t) \neq 0 \quad \text{and} \quad A(t) - H(t) = -\alpha B(t), \qquad G(t) = \beta H(t). \tag{1.4}$$

Random waves are an important subject of stochastic partial differential equations. Konotop and Vázquez investigated nonlinear random waves in [4]. In [2], Holden *et al* gave a white noise functional approach to study stochastic partial differential equations in Wick versions. Xie investigated exact solutions for Wick-type stochastic wave equations in [7–11]. For variable coefficient equations, Zhou *et al* studied the periodic wave solutions for a coupled KdV equation in [12] and gave the homogeneous balance principle and F-expansion method in the same reference. Like Zhou *et al* [12] and Xie [7–11], we shall give exact periodic wave solutions for coupled KP equations with variable coefficients (1.1) with (1.2) and for Wick-type stochastic coupled KP equations (1.3) with (1.4) by using white noise analysis, the homogeneous balance principle and the F-expansion method.

2. SPDEs driven by white noise

In this section, we summarize the main concepts of a white noise functional approach to stochastic partial differential equations. Please see the book of Holden *et al* [2] for details.

Let $h_n(x)$ be the nth Hermite polynomials. Put $\xi_n(x) = \mathrm{e}^{-\frac{1}{2}x^2} h_n(\sqrt{2}x)/(\pi(n-1)!)^{1/2}$, $n \geqslant 1$. We have that the collection $\{\xi_n\}_{n\geqslant 1}$ constitutes an orthogonal basis for $L^2(\mathbb{R})$.

If we denote $\alpha = (\alpha_1, \ldots, \alpha_d)$ being d-dimensional multi-indices by $\alpha_1, \ldots, \alpha_d \in \mathbb{N}$, we have that the family of tensor products $\xi_\alpha = \xi_{(\alpha_1,\ldots,\alpha_d)} = \xi_{\alpha_1} \otimes \ldots \otimes \xi_{\alpha_d}$ $(\alpha \in \mathbb{N}^d)$ forms an orthogonal basis of $L^2(\mathbb{R}^d)$. Let $\alpha^{(i)} = \left(\alpha_1^{(i)}, \ldots, \alpha_d^{(i)}\right)$ be the ith multi-index number in some fixed ordering of all d-dimensional multi-indices $\alpha = (\alpha_1, \ldots, \alpha_d) \in \mathbb{N}^d$. We can, and will, assume that this ordering has the property that

$$i < j \Rightarrow \alpha_1^{(i)} + \cdots + \alpha_d^{(i)} \leqslant \alpha_1^{(j)} + \ldots + \alpha_d^{(j)}.$$

Now define

$$\eta_i = \xi_{\alpha^{(i)}} = \xi_{\alpha_1^{(i)}} \otimes \cdots \otimes \xi_{\alpha_d^{(i)}}, \qquad i \geqslant 1.$$

We denote by $\left(\mathbb{N}_0^{\mathbb{N}}\right)_c$ the space of all multi-indices $\alpha = (\alpha_1, \alpha_2, \ldots)$ with compact support, that is, $\alpha_i \in \mathbb{N}_0$ with finitely many $\alpha_i \neq 0$. We set $\mathcal{J} = \left(\mathbb{N}_0^{\mathbb{N}}\right)_c$. For $\alpha = (\alpha_1, \alpha_2, \ldots) \in \mathcal{J}$, we define

$$H_\alpha(\omega) = \prod_{i=1}^{\infty} h_{\alpha_i}(\langle \omega, \eta_i \rangle), \qquad \omega \in \mathcal{S}'(\mathbb{R}^d).$$

Given $n \in \mathbb{N}$, let $(\mathcal{S})_1^n$ consist of those $x = \sum_\alpha c_\alpha H_\alpha \in \bigoplus_{k=1}^n L^2(\mu)$ with $c_\alpha \in \mathbb{R}^n$ such that $\|x\|_{1,k}^2 = \sum_\alpha c_\alpha^2 (\alpha!)^2 (2\mathbb{N})^{k\alpha} < \infty, \forall k \in \mathbb{N}$ with $c_\alpha^2 = |c_\alpha|^2 = \sum_{k=1}^n \left(c_\alpha^{(k)}\right)^2$ if $c_\alpha = \left(c_\alpha^{(1)}, \ldots, c_\alpha^{(n)}\right) \in \mathbb{R}^n$, where μ is the white noise measure on $(\mathcal{S}'(\mathbb{R}^d), \mathcal{B}(\mathcal{S}'(\mathbb{R}^d)))$, $\alpha! = \prod_{k=1}^\infty \alpha_k!$ and $(2\mathbb{N})^\alpha = \prod_j (2j)^{\alpha_j}$ for $\alpha = (\alpha_1, \alpha_2, \ldots) \in \mathcal{J}$. $\mathcal{S}(\mathbb{R}^d)$, resp. $\mathcal{S}'(\mathbb{R}^d)$, is the Schwartz space, resp. Schwartz distribution space, on $\mathbb{R}^d$.

The space $(\mathcal{S})_{-1}^n$ consists of all formal expansions $X = \sum_\alpha b_\alpha H_\alpha$ with $b_\alpha \in \mathbb{R}^n$ such that $\|X\|_{-1,-q} = \sum_\alpha b_\alpha^2 (2\mathbb{N})^{-q\alpha} < \infty$ for some $q \in \mathbb{N}$. The family of seminorms $\|x\|_{1,k}, k \in \mathbb{N}$ gives rise to a topology on $(\mathcal{S})_1^n$, and we can regard $(\mathcal{S})_{-1}^n$ as the dual of $(\mathcal{S})_1^n$ by the action

$$\langle X, x \rangle = \sum_\alpha (b_\alpha, c_\alpha)\alpha!$$

and (b_α, c_α) is the usual inner product in $\mathbb{R}^n$. $(\mathcal{S})_1^n$, resp. $(\mathcal{S})_{-1}^n$, is called the Kondratiev test function space, resp. Kondratiev distribution space.

For $X = \sum_\alpha a_\alpha H_\alpha$, $Y = \sum_\alpha b_\alpha H_\alpha \in (\mathcal{S})_{-1}^n$ with $a_\alpha, b_\alpha \in \mathbb{R}^n$,

$$X \diamond Y = \sum_{\alpha,\beta} (a_\alpha, b_\beta) H_{\alpha+\beta}$$

is called the Wick product of X and Y.

One can prove that the spaces $\mathcal{S}(\mathbb{R}^d)$, $\mathcal{S}'(\mathbb{R}^d)$, $(\mathcal{S})_1$ and $(\mathcal{S})_{-1}$ are closed under the Wick products.

For $X = \sum_\alpha a_\alpha H_\alpha \in (\mathcal{S})_{-1}^n$ with $a_\alpha \in \mathbb{R}^n$, the Hermite transform of X, denoted by $\mathcal{H}(X)$ or $\widetilde{X}$, is defined by

$$\mathcal{H}(X) = \widetilde{X}(z) = \sum_\alpha a_\alpha z^\alpha \in \mathbb{C}^n \qquad \text{(when convergent)},$$

where $z = (z_1, z_2, \ldots) \in \mathbb{C}^{\mathbb{N}}$ (the set of all sequences of complex numbers) and $z^\alpha = z_1^{\alpha_1} z_2^{\alpha_2} \cdots z_n^{\alpha_n} \ldots$ for $\alpha = (\alpha_1, \alpha_2, \ldots) \in \mathcal{J}$. For $X, Y \in (\mathcal{S})_{-1}$, by this definition we have

$$\widetilde{X \diamond Y}(z) = \widetilde{X}(z) \cdot \widetilde{Y}(z)$$

for all z such that $\widetilde{X}(z)$ and $\widetilde{Y}(z)$ exist. The product on the right-hand side of the above formula is the complex bilinear product between two elements of $\mathbb{C}^N$ defined by $\left(z_1^1, \ldots, z_n^1\right) \cdot \left(z_1^2, \ldots, z_n^2\right) = \sum_{k=1}^n z_k^1 z_k^2$, where $z_k^i \in \mathbb{C}$.

Let $X = \sum_\alpha a_\alpha H_\alpha \in (\mathcal{S})_{-1}^n$. Then the vector $c_0 = \widetilde{X}(0) \in \mathbb{R}^n$ is called the generalized expectation of X and is denoted by $E(X)$. Suppose that $f : V \to \mathbb{C}^m$ is an analytic function, where V is a neighbourhood of $E(X)$. Assume that the Taylor series of f around $E(X)$ has coefficients in $\mathbb{R}^n$. Then the Wick version of f in X, denoted by $f^\diamond(X)$, is defined by $f^\diamond(X) = \mathcal{H}^{-1}(f \circ \widetilde{X}) \in (\mathcal{S})_{-1}^m$.

The Wick exponential of $X \in (\mathcal{S})_{-1}$ can be represented as $\exp^\diamond\{X\} = \sum_{n=0}^\infty X^{\diamond n}/n!$. Using the Hermite transform we have that the Wick exponential has the same algebraic properties as the usual exponential. For example, $\exp^\diamond\{X + Y\} = \exp^\diamond\{X\} \diamond \exp^\diamond\{Y\}$.

Suppose that modelling considerations lead us to consider an SPDE expressed formally as $A(t, x, \partial_t, \nabla_x, U, \omega) = 0$, where A is some given function, $U = U(t, x, \omega)$ is the unknown (generalized) stochastic process, and where the operators $\partial_t = \frac{\partial}{\partial t}$, $\nabla_x = \left(\frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_d}\right)$ when $x = (x_1, \ldots, x_d) \in \mathbb{R}^d$. First, we interpret all products and all functions in the sense of Wick versions. We indicate this by

$$A^\diamond(t, x, \partial_t, \nabla_x, U, \omega) = 0. \tag{2.1}$$

Secondly, we (formally) take the Hermite transform of (2.1). This turns Wick products into ordinary products (between complex numbers) and the equation takes the form

$$\widetilde{A}(t, x, \partial_t, \nabla_x, \widetilde{U}, z_1, z_2, \ldots) = 0, \tag{2.2}$$

where $\widetilde{U} = \mathcal{H}(U)$ is the Hermite transform of U and $z_1, z_2, \ldots$ are complex numbers. Suppose we can find a solution $u = u(t, x, z)$ of the equation $\widetilde{A}(t, x, \partial_t, \nabla_x, u, z) = 0$ for each $z = (z_1, z_2, \ldots) \in \mathbb{K}_q(r)$ for some q, r, where $\mathbb{K}_q(r) = \{z = (z_1, z_2, \ldots) \in \mathbb{C}^{\mathbb{N}}$ and $\sum_{\alpha \neq 0} |z^\alpha|^2 (2\mathbb{N})^{q\alpha} < r^2\}$. Then, under certain conditions, we can take the inverse Hermite transform $U = \mathcal{H}^{-1} u \in (\mathcal{S})_{-1}$ and thereby obtain a solution U of the original Wick equation (2.1). We have the following theorem, which was proved in Holden *et al* [2].

Theorem 2.1. *Suppose $u(t, x, z)$ is a solution (in the usual strong, pointwise sense) of equation (2.2) for (t, x) in some bounded open set $\mathbf{G} \subset \mathbb{R} \times \mathbb{R}^d$, and for all $z \in \mathbb{K}_q(r)$, for* *some q, r. Moreover, suppose that $u(t, x, z)$ and all its partial derivatives, which occur in (2.2), are bounded for $(t, x, z) \in \mathbf{G} \times \mathbb{K}_q(r)$, continuous with respect to $(t, x) \in \mathbf{G}$ for all $z \in \mathbb{K}_q(r)$ and analytic with respect to $z \in \mathbb{K}_q(r)$, for all $(t, x) \in \mathbf{G}$.*

Then there exists a unique $U(t, x) \in (\mathcal{S})_{-1}$ such that $u(t, x, z) = \widetilde{U}(t, x)(z)$ for all $(t, x, z) \in \mathbf{G} \times \mathbb{K}_q(r)$ and $U(t, x)$ solves (in the strong sense in $(\mathcal{S})_{-1}$) equation (2.1) in $(\mathcal{S})_{-1}$.

3. Exact solutions of stochastic coupled KP equations

In this section, we will use theorem 2.1 for $d = 1$ to give exact solutions of (1.1) with (1.2) and (1.3) with (1.4).

Taking the Hermite transform of (1.3), we get the deterministic equations

$$\begin{cases} [\widetilde{U}_t(t, x, z) + \widetilde{A}(t, z)\widetilde{U}(t, x, z)\widetilde{U}_x(t, x, z) + \widetilde{B}(t, z)\widetilde{V}(t, x, z)\widetilde{V}_x(t, x, z) \\ \qquad - \widetilde{H}(t, z)\widetilde{U}_{x^3}(t, x, z)]_x + \widetilde{G}(t, z)\widetilde{U}_{y^2}(t, x, z) = 0, \\ [\widetilde{V}_t(t, x, z) + \widetilde{H}(t, z)\widetilde{U}(t, x, z)\widetilde{V}_x(t, x, z) - \widetilde{H}(t, z)\widetilde{V}_{x^3}(t, x, z)]_x \\ \qquad + \widetilde{G}(t, z)\widetilde{V}_{y^2}(t, x, z) = 0, \end{cases} \tag{3.1}$$

where $z = (z_1, z_2, \ldots) \in \mathbb{C}^{\mathbb{N}}$ is a parameter. We first solve equation (3.1) with the conditions

$$B(t, z) \neq 0, \qquad H(t, z) \neq 0$$

and (3.2)

$$A(t, z) - H(t, z) = -\alpha B(t, z), \qquad G(t, z) = \beta H(t, z).$$

Put $u(t, x, z) = \widetilde{U}(t, x, z), v(t, x, z) = \widetilde{V}(t, x, z), A(t, z) = \widetilde{A}(t, z), B(t, z) = \widetilde{B}(t, z), G(t, z) = \widetilde{G}(t, z)$ and $H(t, z) = \widetilde{H}(t, z)$. We use the F-expansion method to give the solutions of (3.1). Suppose that the solutions of (3.1) are of the form

$$\begin{cases} u(t, x, y, z) = P(\varphi(t, x, y, z)), \\ v(t, x, y, z) = Q(\varphi(t, x, y, z)), \end{cases} \tag{3.3}$$

where $P(\varphi)$ and $Q(\varphi)$ are functions of φ,

$$\varphi(t, x, y, z) = \lambda x + \theta y + \sigma \int_0^t H(s, z)\, \mathrm{d}s + \mu_0, \tag{3.4}$$

where λ, θ and σ are undetermined parameters, μ_0 is a constant.

Substituting (3.3) and (3.4) into (3.1), we have

$$\begin{cases} \lambda\sigma H(t, z)P'' + \lambda^2 A(t, z)[(P')^2 + PP''] + \lambda^2 B(t, z)[(Q')^2 + QQ''] \\ \qquad + \lambda^4 H(t, z)P^{(4)} + \theta^2 G(t)P'' = 0, \\ \lambda\sigma H(t, z)Q'' + \lambda^2 H(t, z)[P'Q' + PQ''] + \lambda^4 H(t, z)Q^{(4)} + \theta^2 G(t, z)Q'' = 0. \end{cases} \tag{3.5}$$

Considering homogeneous balance between $(P')^2 + PP''$, $(Q')^2 + QQ''$ and $P^{(4)}$ in the first equation of (3.5) as well as $P'Q' + PQ''$ and $G^{(4)}$ in the second equation of (3.5), we can suppose that $P(\cdot)$ and $Q(\cdot)$ can be expressed in the following form:

$$\begin{cases} P(\varphi) = a_0 + a_1\Phi(\varphi) + a_2\Phi^2(\varphi), \qquad a_2 \neq 0, \\ Q(\varphi) = b_0 + b_1\Phi(\varphi) + b_2\Phi^2(\varphi), \qquad b_2 \neq 0, \end{cases} \tag{3.6}$$

where a_0, a_1, a_2, b_0, b_1 and b_2 are constants to be determined later, $\Phi(\cdot)$ satisfies the following equation:

$$[\Phi'(\varphi)]^2 = q_0 + q_1\Phi^2(\varphi) + q_2\Phi^4(\varphi). \tag{3.7}$$

This implies that the following relations hold for $\Phi(\varphi)$:

$$\begin{cases} \Phi'(\varphi)\Phi''(\varphi) = q_1\Phi(\varphi)\Phi'(\varphi) + 2q_2\Phi(\varphi)^3\Phi'(\varphi), \\ \Phi''(\varphi) = q_1\Phi(\varphi) + 2q_2\Phi(\varphi)^3, \\ \Phi^{(3)}(\varphi) = q_1\Phi'(\varphi) + 6q_2\Phi(\varphi)^2\Phi'(\varphi) \end{cases} \tag{3.8}$$

where $\varphi(t, x, y, z)$ is defined by (3.4).

Substituting (3.6) and (3.7) into (3.5) and using relations (3.8) yield

$$\begin{aligned} &2\lambda^2\big[-12a_2q_2\lambda^2 H(t,z) + b_2^2 B(t,z) + a_2^2 A(t,z)\big]\Phi^3\Phi' \\ &\quad + 3\lambda^2\big[-2a_1q_2\lambda^2 H(t,z) + a_1a_2A(t,z) + b_1b_2B(t,z)\big]\Phi^2\Phi' \\ &\quad + \big[2a_2\sigma\lambda H(t,z) + b_1^2\lambda^2 B(t,z) - 8a_2q_1\lambda^4 H(t,z) + 2a_0a_2\lambda^2 A(t,z) \\ &\quad + a_1^2\lambda^2 A(t,z) + 2b_0b_2\lambda^2 B(t,z) + 2a_2\theta^2 G(t,z)\big]\Phi\Phi' \\ &\quad + \big[a_1\sigma\lambda H(t,z) + a_0a_1\lambda^2 A(t,z) + b_0b_1\lambda^2 B(t,z) \\ &\quad - a_1q_1\lambda^4 H(t,z) + a_1\theta^2 G(t,z)\big]\Phi' = 0, \end{aligned} \tag{3.9}$$

$$\begin{aligned} &2b_2\lambda^2[a_2 - 12q_2\lambda^2]H(t,z)\Phi^3\Phi' + \lambda^2[2a_1b_2 - 6b_1q_2\lambda^2 + a_2b_1]H(t,z)\Phi^2\Phi' \\ &\quad + [(2b_2\lambda\sigma + 2a_0b_2\lambda^2 - 8b_2q_1\lambda^4 + a_1b_1\lambda^2)H(t,z) + 2b_2\theta^2 G(t,z)]\Phi\Phi' \\ &\quad + b_1[(\lambda\sigma + a_0\lambda^2 - q_1\lambda^4)H(t,z) + \theta^2 G(t,z)]\Phi' = 0. \end{aligned} \tag{3.10}$$

Cancelling Φ' and setting each coefficient of Φ^j $(j = 1, 2, 3)$ to zero implies a system of equations for $a_0, a_1, a_2, b_0, b_1, b_2, \lambda, \theta$ and σ

$$\begin{cases} \lambda^2\big[b_2^2 B(t,z) - 12a_2q_2\lambda^2 H(t,z) + a_2^2 A(t,z)\big] = 0, \\ \lambda^2[a_1a_2A(t,z) - 2a_1q_2\lambda^2 H(t,z) + b_1b_2B(t,z)] = 0, \\ 2a_2\sigma\lambda H(t,z) + b_1^2\lambda^2 B(t,z) - 8a_2q_1\lambda^4 H(t,z) + 2a_0a_2\lambda^2 A(t,z) \\ \qquad + a_1^2\lambda^2 A(t,z) + 2b_0b_2\lambda^2 B(t,z) + 2a_2\theta^2 G(t,z) = 0, \\ a_1\sigma\lambda H(t,z) + a_0a_1\lambda^2 A(t,z) + b_0b_1\lambda^2 B(t,z) - a_1q_1\lambda^4 H(t,z) + a_1\theta^2 G(t,z) = 0, \\ 2b_2\lambda^2[a_2 - 12q_2\lambda^2]H(t,z) = 0, \\ \lambda^2[2a_1b_2 - 6b_1q_2\lambda^2 + a_2b_1]H(t,z) = 0, \\ (2b_2\lambda\sigma + 2a_0b_2\lambda^2 - 8b_2q_1\lambda^4 + a_1b_1\lambda^2)H(t,z) + 2b_2\theta^2 G(t,z) = 0, \\ b_1[(\lambda\sigma + a_0\lambda^2 + q_1\lambda^4)H(t,z) + \theta^2 G(t,z)] = 0. \end{cases} \tag{3.11}$$

The solutions of (3.11) under the condition (3.2) and $a_2b_2\lambda\theta\sigma \neq 0$ are

$$\begin{cases} a_0 = \pm\sqrt{\alpha}b_0, \qquad a_1 = 0, \qquad b_1 = 0, \\ a_2 = 12\lambda^2 q_2, \qquad b_2 = \pm 12\lambda^2 q_2\sqrt{\alpha}, \\ \theta^2 = \frac{\lambda^2}{\beta}(4\lambda^2 q_1 \mp \sqrt{\alpha}b_0) - \frac{\lambda}{\beta}\sigma, \\ \lambda, \sigma, \beta, b_0, q_1 \text{ and } q_2 \text{ are arbitrary constants.} \end{cases} \tag{3.12}$$

Substituting (3.12) into (3.6) implies that, for any $z \in \mathbb{C}^{\mathbb{N}}$, the general form solutions of (3.1) with (3.2) are the following:

$$\begin{cases} u(t,x,y,z) = P(\varphi(t,x,y,z)) = \pm\sqrt{\alpha}b_0 + 12\lambda^2 q_2\Phi^2(\varphi(t,x,y,z)), \\ v(t,x,y,z) = Q(\varphi(t,x,y,z)) = b_0 \pm 12\lambda^2 q_2\sqrt{\alpha}\Phi^2(\varphi(t,x,y,z)) \end{cases} \tag{3.13}$$

with $\varphi(t, x, y, z)$ being defined by (3.4), θ, σ and λ satisfy (3.12); $\alpha > 0$, b_0, q_1, q_2 and μ_0 are arbitrary constants.

If we can prove that there exist a bounded open set $\mathbf{G} \subset \mathbb{R}_+ \times \mathbb{R}^2$, $q > 0$ and $r > 0$ such that $u(t,x,y,z)$, $u_{tx}(t,x,y,z)$, $u_x(t,x,y,z)$, $u_{x^2}(t,x,y,z)$, $u_{x^4}(t,x,y,z)$, $u_{y^2}(t,x,y,z)$, $v(t,x,y,z)$, $v_{tx}(t,x,y,z)$, $v_{x^2}(t,x,y,z)$, $v_{x^4}(t,x,y,z)$ and $v_{y^2}(t,x,y,z)$ are uniformly bounded for all $(t,x,y,z) \in \mathbf{G} \times \mathbb{K}_q(r)$, continuous with respect to $(t,x,y) \in \mathbf{G}$ for all $z \in \mathbb{K}_q(r)$ and analytic with respect to $z \in \mathbb{K}_q(r)$ for all $(t,x,y) \in \mathbf{G}$, theorem 2.1 will show that there exist $U(t,x,y), V(t,x,y) \in (\mathcal{S})_{-1}$ such that $u(t,x,y,z) = \mathcal{H}U(t,x,y)(z)$ and $v(t,x,y,z) = \mathcal{H}V(t,x,y)(z)$ for all $(t,x,y,z) \in \mathbf{G} \times \mathbb{K}_q(r)$ and that $U(t,x,y)$ and $V(t,x,y)$ solve (1.3) with (1.4). From the above, we have that $U(t,x,y)$ and $V(t,x,y)$ are the inverse Hermite transformations of $u(t,x,y,z)$ and $v(t,x,y,z)$. Hence, (3.13) yields that solutions of (1.3) with (1.4) are

$$\begin{cases} U(t,x,y) = \pm\sqrt{\alpha}b_0 + 12\lambda^2 q_2 \Phi^{\diamond 2}(\Psi(t,x,y)), \\ V(t,x,y) = b_0 \pm 12\lambda^2 q_2 \sqrt{\alpha}\Phi^{\diamond 2}(\Psi(t,x,y)) \end{cases} \tag{3.14}$$

with

$$\Psi(t,x,y) = \lambda x + \theta y + \sigma \int_0^t H(s)\,\mathrm{d}s + \mu_0, \tag{3.15}$$

where θ, σ and λ satisfy (3.12), $\alpha > 0$, b_0, q_1, q_2 and μ_0 are arbitrary constants.

For different parameters q_0, q_1 and q_2, table 1 of Zhou *et al* in [12] shows the solutions for (3.7). Hence, we shall use table 1 of Zhou *et al* and (3.7) to give two examples for different q_0, q_1 and q_2.

(i) For $q_0 = 1, q_1 = -1 - m^2$ and $q_2 = m^2$, the solution of (3.7) is $\Phi(\varphi(t,x,y,z)) = \operatorname{sn}\varphi(t,x,y,z)$. Thus, the solutions of (3.1) with (3.2) are

$$\begin{cases} u(t,x,y,z) = F(\varphi(t,x,y,z)) = \pm\sqrt{\alpha}b_0 + 12\lambda^2 m^2 \operatorname{sn}^2\varphi(t,x,y,z), \\ v(t,x,y,z) = G(\varphi(t,x,y,z)) = b_0 \pm 12\lambda^2 m^2 \sqrt{\alpha}\operatorname{sn}^2\varphi(t,x,y,z) \end{cases} \tag{3.16}$$

with $\varphi(t,x,y,z)$ being defined by (3.4).

The property of the Jacobian elliptic function $\operatorname{sn} x$ (see chapter 10 of Wang and Guo [6] for details) shows that the conditions of theorem 2.1 are all satisfied. Hence, there are stochastic processes $U(t,x,y)$ and $V(t,x,y)$ which are the inverse Hermite transformations of $u(t,x,y,z)$ and $v(t,x,y,z)$ such that

$$\begin{cases} U(t,x,y) = \pm\sqrt{\alpha}b_0 + 12\lambda^2 m^2 \operatorname{sn}^{\diamond 2}\Psi(t,x,y), \\ V(t,x,y) = b_0 \pm 12\lambda^2 m^2 \sqrt{\alpha}\operatorname{sn}^{\diamond 2}\Psi(t,x,y) \end{cases} \tag{3.17}$$

are solutions of (1.3) with (1.4), where $\Psi(t,x,y)$ is defined by (3.15).

Put $m \to 1$, we have $\operatorname{sn}\varphi \to \tanh\varphi$. Thus, (3.16) becomes

$$\begin{cases} u(t,x,y,z) &= \pm\sqrt{\alpha}b_0 + 12\lambda^2 \tanh^2\varphi(t,x,y,z) \\ &= \pm\sqrt{\alpha}b_0 + 12\lambda^2 - 12k\lambda^2 \operatorname{sech}^2\varphi(t,x,y,z), \\ v(t,x,y,z) &= b_0 \pm 12\lambda^2\sqrt{\alpha}\tanh^2\varphi(t,x,y,z) \\ &= b_0 \pm 12\lambda^2\sqrt{\alpha} \mp 12\lambda^2\sqrt{\alpha}\operatorname{sech}^2\varphi(t,x,y,z) \end{cases} \tag{3.18}$$

with $\varphi(t,x,y,z)$ as (3.4). Equation (3.18) gives the solitary wave solutions of (3.1) with (3.2) for any fixed $z \in \mathbb{C}^{\mathbb{N}}$.

Using the property of the function $\operatorname{sech} x$ and (3.18), we have

$$\begin{cases} U(t,x,y) = \pm\sqrt{\alpha}b_0 + 12\lambda^2 - 12k\lambda^2 \operatorname{sech}^{\diamond 2}\Psi(t,x,y), \\ V(t,x,y) = b_0 \pm 12\lambda^2\sqrt{\alpha} \mp 12\lambda^2\sqrt{\alpha}\operatorname{sech}^{\diamond 2}\Psi(t,x,y) \end{cases} \tag{3.19}$$

are solutions of (1.3) with (1.4), where $\Psi(t,x,y)$ is defined by (3.15).

Choosing $H(t) = f(t) + W_t$, where $f(t)$ is a real bounded or integrable function on $\mathbb{R}_+$, $W_t = \dot{B}_t$ and B_t is a Brownian motion, we have that $U(t, x, y)$ and $V(t, x, y)$ satisfy (3.17) or (3.19) (for $m \to 1$) with

$$\Psi(t, x, y) = \lambda x + \theta y + \sigma \left[\int_0^t f(s)\,\mathrm{d}s + B_t \right] + \mu_0. \tag{3.20}$$

Since $\exp^\diamond\{B_t\} = \exp\left\{B_t - \frac{1}{2}t^2\right\}$, (3.19), (3.20) and the definition of the function sech x yield that the solutions of (1.3) with (1.4) are

$$\begin{cases} U(t, x, y) = \pm\sqrt{\alpha} b_0 + 12\lambda^2 - 12k\lambda^2 \operatorname{sech}^2 \Psi_1(t, x, y), \\ V(t, x, y) = b_0 \pm 12\lambda^2\sqrt{\alpha} \mp 12\lambda^2\sqrt{\alpha} \operatorname{sech}^2 \Psi_1(t, x, y) \end{cases} \tag{3.21}$$

with

$$\Psi_1(t, x, y) = \lambda x + \theta y + \sigma \left[\int_0^t f(s)\,\mathrm{d}s + B_t - \frac{1}{2}t^2 \right] + \mu_0. \tag{3.22}$$

In (3.16)–(3.22), θ, σ and λ satisfy $\beta\theta^2 + \lambda\sigma + \lambda^2[\lambda^2(1+m) \pm \sqrt{\alpha} b_0] = 0$, $\alpha > 0$, β, μ_0 and b_0 are arbitrary constants.

(ii) For $q_0 = 1 - m^2$, $q_1 = 2m^2 - 1$ and $q_2 = -m^2$ $(0 < m < 1)$, we have $\Phi(\varphi) = \operatorname{cn}\varphi$ and

$$\begin{cases} u(t, x, y, z) = \pm\sqrt{\alpha} b_0 - 12\lambda^2 m^2 \operatorname{cn}^2 \varphi(t, x, y, z), \\ v(t, x, z) = b_0 \mp 12\lambda^2 m^2 \sqrt{\alpha} \operatorname{cn}^2 \varphi(t, x, y, z), \end{cases} \tag{3.23}$$

with $\varphi(t, x, y, z)$ which is defined by (3.4), are solutions of (3.1) with (3.2).

The property of the Jacobian elliptic function cn x (see chapter 10 of Wang and Guo [6] for details) shows that the conditions of theorem 2.1 are also satisfied. Hence, there are stochastic processes $U(t, x, y)$ and $V(t, x, y)$ which are white noise functionals and are the inverse Hermite transformations of $u(t, x, y, z)$ and $v(t, x, y, z)$ such that

$$\begin{cases} U(t, x, y) = \pm\sqrt{\alpha} b_0 - 12\lambda^2 m^2 \operatorname{cn}^{\diamond 2} \Psi(t, x, y), \\ V(t, x, y) = b_0 \mp 12\lambda^2 m^2 \sqrt{\alpha} \operatorname{cn}^{\diamond 2} \Psi(t, x, y), \end{cases} \tag{3.24}$$

with $\Psi(t, x, y)$ being defined by (3.15), are solutions of (1.3) with (1.4).

For $m \to 1$, we have $\operatorname{cn}\varphi \to \operatorname{sech}\varphi$ and (3.23) yields

$$\begin{cases} u(t, x, y, z) = \pm\sqrt{\alpha} b_0 - 12\lambda^2 \operatorname{sech}^2 \varphi(t, x, y, z), \\ v(t, x, y, z) = b_0 \mp 12\lambda^2 \sqrt{\alpha} \operatorname{sech}^2 \varphi(t, x, y, z), \end{cases} \tag{3.25}$$

with $\varphi(t, x, y, z)$ being defined by (3.4), are solutions of (3.1) with (3.2).

Equation (3.25) shows that solutions of (1.3) with (1.4) are

$$\begin{cases} U(t, x, y) = \pm\sqrt{\alpha} b_0 - 12\lambda^2 \operatorname{sech}^{\diamond 2} \Psi(t, x, y), \\ V(t, x, y) = b_0 \mp 12\lambda^2 \sqrt{\alpha} \operatorname{sech}^{\diamond 2} \Psi(t, x, y), \end{cases} \tag{3.26}$$

with $\Psi(t, x, y)$ being defined by (3.15).

For $H(t) = f(t) + W_t$, as in the discussion in case (i) we have that solutions of (1.3) with (1.4) are the following:

$$\begin{cases} U(t, x, y) = \pm\sqrt{\alpha} b_0 - 12\lambda^2 \operatorname{sech}^2 \Psi_1(t, x, y), \\ V(t, x, y) = b_0 \mp 12\lambda^2 \sqrt{\alpha} \operatorname{sech}^2 \Psi_1(t, x, y), \end{cases} \tag{3.27}$$

with $\Psi_1(t, x, y)$ being defined by (3.22).

In (3.23)–(3.27), θ, σ and λ satisfy $\beta\theta^2 + \lambda\sigma - \lambda^2[4\lambda^2(2m^2 - 1) \mp \sqrt{\alpha} b_0] = 0$, $\alpha > 0$, β, μ_0 and b_0 are arbitrary constants.

Remark.

(i) Since $f^{\diamond}(t) = f(t)$ for any nonrandom measurable function $f(t)$, (3.14), (3.17), (3.19), (3.24) and (3.26) give the solutions of the coupled KP equations with variable coefficients (1.1) with (1.2).

(ii) Since there is a unitary map between the Wiener white noise space and the Poisson white noise space, we can obtain solutions of the Poissonian SPDEs simply by applying this map to solutions of the corresponding Gaussian SPDEs. A nice, concise account of this connection was given by Benth and Gjerde in [1] and by Holden *et al* in section 4.9 of [2]. Hence, we can get exact periodic wave solutions of (1.3) in the same way as in the Gaussian case, when $A(t)$, $B(t)$, $H(t)$ and $G(t)$ are Poissonian white noise functionals, satisfying (1.4).

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